Signal and Data Analysis

Exercise 8

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1 Karnaugh Maps

Boolean functions can be expressed as a sum of products (SoP), where the sum of Boolean variables is their logical OR and the product is the logical AND. Karnaugh maps are a useful tool construct a minimal SoP for a Boolean function of up to 6 variables. The Karnaugh map is the representation of a Boolean function as a two-dimensional grid. The size of the grid is 2×2 for a function of two variables, 4×2 for three and 4×4 for four variables. Between neighboring rows and columns, only one variable is allowed to change.

A	B	C	D	f(A,B,C,D)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

AB

CD	00	01	11	10
00				
01				
11				
10				

The first step is to transfer the given truth table to the Karnaugh map. Then rectangles are located that contain only 1s and that have a number of fields that is a power of 2 (1,2,4. . .). Each of the rectangles can be expressed as a product of the Boolean variables or their negations. The minimal set of rectangles that are as large as possible is the minimal SoP representing the Boolean function.

(a) Convert the truth table given above into a Karnaugh map.

(b) Find a set of rectancles that covers all 1s in the map.

(c) Translate the rectangular areas into Boolean expressions of the four input variables and give the minimal sum of products of the function f.

2 Floating Point Numbers

In digital systems, non-integer real numbers are often represented as *floating point numbers*:

$$x = s \cdot b^e$$

where s is called the significand or mantissa, b the base and e the exponent. For binary representation, the base $b = 2$. The floating point representation is normalized if the significand s has only a single digit left of the radix point, i.e. $1 \leq s < b$.
The standard IEEE-754 defines floating point representations in single and double precision. A single precision number uses 24 bits for the normalized significand, 8 bits for the exponent and one sign bit. The exponent can take values from -126 to $+127$, using 254 of the 256 bit patterns while the other two are reserved for marking special cases. A double precision number uses a 53-bit significand and an 11-bit exponent for values from -1022 to $+1023$. As the digit of the mantissa is always 1, it can be omitted and the total size is $23 + 8 + 1 = 32$ bits for a single and $52 + 11 + 1 = 64$ bits for a double precision number.

(a) Convert the number π to a binary floating point number. Give the significand with a precision of 8 binary digits.

(b) What are the largest and the smallest non-zero positive numbers that a single respectively double precision floating point number can take? What is the smallest number greater than 1?

(c) A floating point system contains many but not all integer numbers. Find the smallest integers that do not have an exact representation in single and double precision floating point numbers.

(d) Write a program to confirm the result from (c).